Wind Shear near the Ground and Aircraft Operations

GEORGE H. FICHTL*
NASA Marshall Space Flight Center, Huntsville, Ala.

The variance of wind shear in the first 150–200 m of the atmosphere is a function of the direction of the mean wind relative to the flight path, the zenith angle of the flight path, the standard deviation of the three components of the turbulence velocity vector, the surface friction velocity, the stability properties of theatmospheric boundary layer, and the heights above natural grade of the beginning and end points of the portion of the flight path over which the shear is to be calculated. The results are applied by calculating wind shear for various risks of occurrence assuming wind shear is a Gaussian process, and it is shown that turbulence produces significantly large dispersions in wind shear about the mean wind shear. The results are interpreted in terms of the ICAO interim shear criteria for reporting wind shear in qualitative terms.

Introduction

THIS paper is concerned with the statistical properties of wind shear in the first 150–200 m of the planetary boundary layer (heights $\lesssim 1$ km). In this paper wind shear is defined to be the variation of the component of wind along the flight path of an airplane. As an airplane ascends or descends through the boundary layer it will encounter wind shear. This will result in a sudden increase or decrease in aerodynamic lift force depending on whether or not the shear corresponds to an increase or decrease of the wind speed along the flight path. The shear produces a near-instantaneous change in the lift force[1] to which the aircraft and pilot take a finite time to respond. Accordingly, when shear is encountered in the boundary layer the airplane will respond by accelerating vertically away from the flight path. The pilot will then respond to the acceleration and attempt to bring the plane back to the desired flight path. It is therefore obvious that wind shear could have important implications with regard to takeoff and particularly landing operations. A number of authors[2] have cited wind shear as being responsible for unanticipated long and short touchdowns.

Wind shear is composed of two parts, namely, mean wind shear and fluctuating shear. The mean shear is produced by the mean wind profile, whereas the fluctuating shear is produced by the action of atmospheric turbulence. A number of investigations concerning mean wind shear have been conducted. However, only a relatively small number of investigations have been concerned with fluctuating shear. It is the intent of this paper to examine both the mean and fluctuating shear with emphasis on the latter.

Total Wind Shear

In the atmospheric boundary layer the wind vector $\mathbf{V}(x,y,z,t)$ at horizontal position (x,y) height z, and time t can be represented as the sum of a mean flow velocity vector $\mathbf{V}_0(x,y,z,t)$ and a superimposed turbulent velocity fluctuation. If we restrict our analysis to the case of the horizontally homogeneous and stationary boundary layer, then the statistics of the flow (mean flow velocity, variance of turbulence, etc.) do not depend on the horizontal position (x,y) or time t, so that the statistics can only vary in the vertical along the z-axis. This condition is rarely realized in the atmospheric boundary layer in the strictest sense because the surface roughness and heat-transfer properties of the Earth vary in the horizontal. However, as an engineering approximation, the concept of the horizontally homogeneous and stationary boundary layer appears to be applicable for those situations in which the terrain is relatively flat. More will be said about this point later.

In the horizontally homogeneous and stationary boundary layer the mean wind velocity is a function of z only and the vertical component of the mean wind velocity vanishes. Furthermore, we are interested only in the first 150–200 m of the boundary layer. In this region of the atmosphere the mean wind direction has small vertical variation. Thus, upon invoking these restrictions we may express the velocity vector of the air as

$$\mathbf{V} = [\bar{u}(z) + u'(x,y,z,t)]\mathbf{i} + v'(x,y,z,t)\mathbf{j} + w'(x,y,z,t)\mathbf{k} \quad (1)$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors directed along the x, y, z axes; $\bar{u}(z)$ is a mean wind speed at height z (the mean wind is directed along the x axis); and u', v', and w' are the x, y, z components of the turbulent velocity fluctuation vector. In theory, the quantity $\bar{u}(z)$ is an ensemble mean wind. However, we cannot evaluate the ensemble mean because an extremely large number of realizations of the turbulent flow with the same external conditions are not available. Accordingly, we invoke the ergodic hypothesis and replace the ensemble mean with a time average over a given realization of turbulence. The time average of the quantity $\xi(t)$ over time interval $0 \leq t \leq t_0$ is given by

$$\bar{\xi} = \frac{1}{t_0}\int_0^{t_0} \xi(t)dt \quad (2)$$

A time average over a period of 10 min to 1 hr usually results in stable estimates of statistics such as the mean or variance. It should be noted that if we time average Eq. (1) then $\mathbf{V}_0 = \bar{u}(z)\mathbf{i}$, so that $\bar{u}' = \bar{v}' = \bar{w}' = 0$. Henceforth, it will be understood that all ensemble averages are to be estimated with time averages.

A unit vector $\mathbf{n}$ along the flight path of an aircraft during take off or landing is given by

$$\mathbf{n} = \sin\gamma \cos\theta\mathbf{i} + \sin\gamma \sin\theta\mathbf{j} + \cos\gamma\mathbf{k} \quad (3)$$

where γ is the angle between the flight path and the z-axis, and θ is the angle between the horizontal projection of the flight path and the mean wind vector.

Upon forming the scalar product between Eq. (1) and Eq. (3), we find the projection of the velocity vector of the air onto the flight path to be given by

$$V_f = [\bar{u}(z) + u'] \sin\gamma \cos\theta + v' \sin\gamma \sin\theta + w' \cos\gamma \quad (4)$$

Now the change ΔV_f of the component of the wind along the flight path between points (x_1,y_1,z_1,t_1) and (x_2,y_2,z_2,t_2) can be

Received January 19, 1972; revision received August 10, 1972.
Index categories: Aircraft Gust Loading and Wind Shear; VTOL Handling, Stability, and Control.
* Scientific Assistant.

obtained by evaluating Eq. (4) at these points and subtracting the resulting relationship to yield

$$\Delta V_f = (\Delta\bar{u} + \Delta u')\sin\gamma\cos\theta + \Delta v'\sin\gamma\sin\theta + \Delta w'\cos\gamma \quad (5)$$

where

$$\Delta\bar{u} = \bar{u}(z_2) - \bar{u}(z_1)$$
$$\Delta u' = u'(x_2,y_2,z_2,t_2) - u'(x_1,y_1,z_1,t_1)$$
$$\Delta v' = v'(x_2,y_2,z_2,t_2) - v'(x_1,y_1z_1,t_1)$$
$$\Delta w' = w'(x_2,y_2,z_2,t) - v'(x_1,y_1,z_1t)$$

Eq. (5) is the total wind shear between points (x_1,y_1,z_1,t_1) and (x_2,y_2,z_2,t_2) along the flight path of an aircraft.

Mean Wind Shear

Upon ensemble averaging Eq. (5) we find the mean wind shear along the flight path to be given by

$$\langle\Delta V_f\rangle = \Delta\bar{u}\sin\gamma\cos\theta \quad (6)$$

where angular brackets denote an ensemble average. In the horizontally homogeneous, fully turbulent and stationary boundary layer, an adequate representation of the mean wind profile is given by

$$\bar{u}(z) = (u_{*0}/k)[\ln(z/z_0) - \psi(z/L_0)] \quad (7)$$

where u_{*0} is the surface friction velocity ($u_{*0}{}^2$ is the tangential stress per unit mass at the surface of the Earth), z_0 is the surface roughness length, k is von Kármáns constant with numerical value approximately equal to 0.4, and ψ is a universal function[3] of z/L_0. The surface roughness length varies from 10^{-5} m for mud flats and ice to roughly 1–4 m for cities (see Table 1). The quantity L_0 is the surface Monin–Obukhov stability length and is given by

$$L_0 = -\,u_{*0}{}^3 C_p\bar{\rho}\bar{T}/kgH_0 \quad (8)$$

where H_0 is the surface heat flux, $\bar{\rho}$ and $\bar{T}$ denote the mean flow density, Kelvin temperature, g is the acceleration of gravity, and C_p is the specific heat at constant pressure. The quantity L_0 is a stability parameter such that the ratio $-z/L_0$ can be interpreted as being

$$-(z/L_0) \simeq p_b/p_m \quad (9)$$

where p_m and p_b denote the mechanical and buoyant production rates of turbulent kinetic energy. Mechanical production comes about from the shearing action of the mean wind, while buoyant production comes about from heating of the atmosphere adjacent to the ground. Thus, if $-z/L_0 \gtrsim 1$, then buoyant production exceeds mechanical production, and $-z/L_0 \lesssim 1$ corresponds to mechanical production in excess of buoyant production. If $z/L_0 = 0$, then buoyant production vanishes and the turbulence is created mechanically by the mean wind shear. It should be noted that if $z/L_0 > 0$, then $H_0 < 0$, which means there is a downward flux of thermal energy into the ground from the air. In this case, the buoyant production is negative so that the turbulence produced by the mechanical action of the wind shear tends to be damped by the action of negative buoyant forces. Available evidence appears to show that when $z/L_0 \gtrsim 0.18$ turbulence tends to cease. The case of high-wind speeds is one case of engineering interest because the high-wind speed situation will produce large values of mean and fluctuating wind shear. As the wind speed increases, the mechanical production rate or turbulent kinetic energy increases such that at sufficiently high-wind speeds mechanical production will exceed buoyant production. Thus, $z/L_0 \to 0$ as the mean flow wind speed becomes large, so that at sufficiently large wind speeds we have $z/L_0 \simeq 0$. When this condition occurs the function $\psi(z/L_0)$ vanishes and the mean wind profile takes on logarithmic behavior, namely

$$\bar{u}(z) = (u_{*0}/k)\ln(z/z_0) \quad (10)$$

This is the wind profile for the neutral boundary layer.

When the surface is very rough, as in the case of a forest or a city, the vegetation or buildings produce an effective surface above the solid ground. Accordingly, an additional parameter, the zero-plane displacement,[4] d is introduced into Eq. (7). To do this z is replaced with $z - d$ and Eq. (7) becomes

$$\bar{u}(z)(u_{*0}/k)\{\ln[(z-d)/z_0] - \psi[(z-d)/L_0]\} \quad (7a)$$

The parameter d is approximately equal to the typical height of the vegetation and/or the buildings and can become quite large. Henceforth, we will use Eq. (7) in our calculations; however, it is understood that z is referenced to the zero plane level (height d above natural grade). Substitution of Eq. (7) into Eq. (6) yields

$$\langle\Delta V_f\rangle/u_{*0} = (\Delta\bar{u}/u_{*0})\sin\gamma\cos\theta \quad (11)$$

where

$$\Delta\bar{u}/u_{*0} = (1/k)\{\ln[(1 + \Delta z/2\bar{z})/(1 - \Delta z/2\bar{z})] + \psi(z_1/L_0) - \psi(z_2/L_0)\} \quad (12)$$

The quantity $\bar{z}$ is the height of the midpoint of the interval Δz above the zeroplane level. Thus, specification of the surface friction velocity, the Monin–Obukhov stability length, the altitude interval Δz over which the shear is to be computed and the height $\bar{z}$ permits the calculation of the mean wind shear $\Delta\bar{u}$. Furthermore, specification of the angles θ and γ permits the calculation of the mean wind shear along the

Table 1 Typical values of surface roughness length (z_0) for various types of surfaces

Type of surface	$z_0(m)$
Mud flats, ice	10^{-5}–$3\cdot10^{-5}$
Smooth sea	2.10^{-4}–$3\cdot10^{-4}$
Sand	10^{-4}–10^{-3}
Snow surface	10^{-3}–$6\cdot10^{-3}$
Mown grass (~0.01 m)	10^{-3}–10^{-2}
Low grass, steppe	10^{-2}–$4\cdot10^{-2}$
Fallow field	2.10^{-2}–$3\cdot10^{-2}$
High grass	4.10^{-2}–10^{-1}
Palmetto	10^{-1}–$3\cdot10^{-1}$
Suburbia	1 –2
City	1 –4

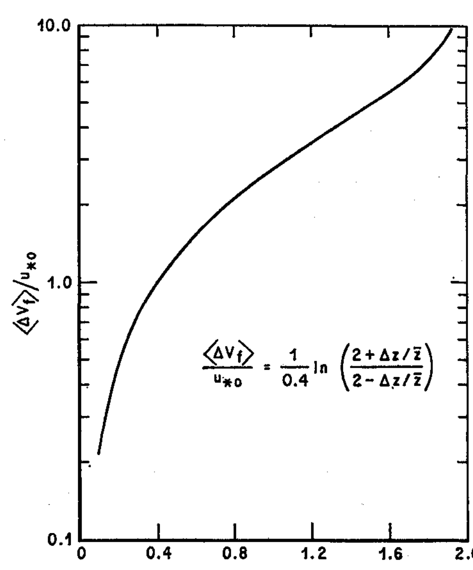

Fig. 1 Nondimensional mean wind shear $\langle\Delta V_f\rangle/u_{*0}$ as a function of nondimensional shear interval $\Delta z/\bar{z}$.

flight path. A plot of the nondimensional mean wind shear $\Delta\bar{u}/u_{*0}$ for neutral wind conditions ($z/L_0 = 0$ and $\psi(0) = 0$) is given in Fig. 1. The function $\psi(z/L_0)$ is a monotonically decreasing function of z/L_0 with $\psi(0) = 0$. This means that in unstable air $0 < \psi(z_1/L_0) < \psi(z_2/L_0)$, so that the curves of $\Delta u/u_{*0}$ as functions of $\Delta z/\bar{z}$ for fixed values of $\bar{z}/L_0 < 0$ fall below the neutral curve in Fig. 1. Similarly, in stable air $\psi(z_2/L_0) < \psi(z_1/L_0) < 0$, so that the stable mean shear curves lie above the neutral shear curve.

Standard Deviation of Wind Shear

Subtraction of Eq. (6) from Eq. (5) yields the fluctuation portion of the shear resulting from atmospheric turbulence, namely,

$$\Delta V_f' = \Delta u' \sin\gamma \cos\theta + \Delta v' \sin\gamma \sin\theta + \Delta w' \cos\gamma \quad (13)$$

A particularly useful measure of the fluctuating shear $\Delta V_f'$ is the standard deviation σ of the shear which can be obtained by squaring both sides of Eq. (13) and ensemble-averaging (really time-averaging) the resulting relationship, so that

$$\begin{aligned}\sigma^2 = {} & \langle(\Delta u')^2\rangle \sin^2\gamma \cos^2\theta + \langle(\Delta v')^2\rangle \sin^2\gamma \sin^2\theta \\ & + \langle(\Delta w')^2\rangle \cos^2\gamma + 2\langle\Delta u'\Delta v'\rangle \sin^2\gamma \cos\theta \sin\theta \\ & + 2\langle\Delta u'\Delta w'\rangle \sin\gamma \cos\gamma \cos\theta \\ & + 2\langle\Delta v'\Delta w'\rangle \sin\gamma \cos\gamma \sin\theta \end{aligned} \quad (14)$$

In the atmospheric boundary layer u' and v', and v' and w' tend to be uncorrelated. Therefore, the forth and sixth terms on the righthand side of Eq. (14) can be set equal to zero. For the sake of simplicity we shall assume that the distance between the two points over which the shear is to be calculated is sufficiently large such that all correlations of velocity components between the points of concern can be neglected. This assumption appears to be reasonable in view of the fact that the integral scale of turbulence in the lower portion (first 150 m) of the boundary layer is approximately 200 m, which is less than or approximately equal to the horizontal distance an airplane must travel during the portion of flight in which wind shear may be important. However, there can be nonzero correlations between components u' and w' at a point. Accordingly, we may write Eq. (14) as

$$\begin{aligned}\sigma^2 = {} & [\sigma_u^2(z+\Delta z) + \sigma_u^2(z)] \sin^2\gamma \cos^2\theta \\ & + [\sigma_v^2(z+\Delta z) + \sigma_v^2(z)] \sin^2\gamma \sin^2\theta \\ & + [\sigma_w^2(z+\Delta z) + \sigma_w^2(z)] \cos^2\gamma \\ & + 2[\langle u'(x_2,y_2,z_2,t_2) w'(x_2,y_2,z_2,t_2)\rangle \\ & + \langle u'(x_1,y_1,z_1,t_1) w'(x_1,y_1,z_1,t_1)\rangle] \sin\gamma \cos\gamma \cos\theta \end{aligned} \quad (15)$$

where $\sigma_u(z)$, $\sigma_v(z)$, and $\sigma_w(z)$ are the standard deviations of u', v', and w' at height z. The quantity $\langle u'w'\rangle$ is the vertical turbulent transport of x directed momentum. To a sufficient degree of approximation, the vertical momentum transport in the lower portion of the boundary layer is equal to the negative of the square of the surface friction velocity, i.e.

$$-\langle u'w'\rangle = u_{*0}^2 \quad (16)$$

Since the mean wind increases with height, this transport is directed downward. The standard deviations of turbulence scaled with the friction velocity are functions of z/L_0. Typical values of the scaled standard deviations for neutral wind conditions ($z/L_0 = 0$) are

$$\sigma_u/u_{*0} = 2.5; \quad \sigma_v/u_{*0} = 2.0; \quad \sigma_w/u_{*0} = 1.3 \quad \text{(see Ref. 5)} \quad (17)$$

The scaled standard deviations tend to increase as z/L_0 decreases away from neutral conditions toward instability ($z/L_0 < 0$) and decrease as z/L_0 increases away from neutral conditions toward stability ($z/L_0 > 0$). We shall base our analysis on the neutral boundary layer. However, the calculations which follow can easily be modified for the nonneutral boundary layers.

Table 2 Interim classification for reporting wind shear in the atmospheric boundary layer according to the ICAO[6]

Category	Shear magnitude, m sec^{-1}/30 m
Light	0–2.5
Moderate	2.5–4.5
Strong	4.5–6.0
Severe	>6

Substitution of Eqs. (16) and (17) into Eq. (15) yields

$$(\sigma/u_{*0})^2 = 2[(2.25 \cos^2\theta + 4) \sin^2\gamma + 1.69 \cos^2\gamma - 2 \sin\gamma \cos\gamma \cos\theta] \quad (18)$$

Thus, an estimate of the friction velocity u_{*0} and the angles θ and γ permits an estimation of the standard deviation of the fluctuating portion of the wind shear along the flight path.

An Application

The international Civil Aviation Organization (ICAO) has made several recommendations concerning wind shear conditions during the landing approach and climb-out phases of flight.[6] Among these recommendations was an interim shear criterion, namely, that vertical wind shear, when reported in qualitative terms, should be expressed according to the classifications given in Table 2. This classification was not officially accepted because it is not universal. For example, moderate shear for a high stall-speed aircraft could correspond to strong shear for a relatively low stall-speed airplane. Nevertheless, the classification of shear in Table 2 gives us estimates of the shear magnitudes that are important to aircraft operations. What is the risk of encountering the shear magnitudes as given in Table 2 for specific boundary-layer conditions? To answer this question, we assume that the nondimensional shear as experienced by the aircraft along the flight path during a given realization of turbulence is distributed according to the Gaussian distribution, whose density function is given by

$$p(\eta) = (2\pi\sigma/u_{*0})^{-1/2} \exp-(\eta - \mu_\eta)^2/2(\sigma/u_{*0})^2 \quad (19)$$

where $\eta = \Delta V_f/u_{*0}$ and μ_η is the average value of η. The assumption that atmospheric turbulence constitutes a Gaussian process has come under considerable scrutiny in the last five years. Many investigators have found that turbulence does not constitute a Guassian process.[7] Recent studies at the NASA Marshall Space Flight Center, which will be reported soon, have shown that as the distance between the points over which the shear is calculated becomes large then the third and fourth moments of wind shear, when standardized with the appropriate powers of the shear standard deviation, approach zero and three, respectively. These values are appropriate for a normal distribution. As the shear interval becomes small, the standardized third and fourth moments become large and depart significantly from their Gaussian values. The shear interval distances of concern in this report are sufficiently large that the third and fourth moments of wind shear have Gaussian-like behavior. In other words, the wind shear statistics for the aircraft landing problem seem to have Gaussian behavior up to and including the fourth moment. This does not mean the higher-order moments will have Gaussian behavior; nevertheless, in view of the forgoing comments, the Gaussian assumption is not an unreasonable one for the engineering application intended here. Integration of Eq. (19) over the interval $-\infty < \eta < \eta^*$ will yield the probability P of $\eta \leq \eta^*$ during a given realization of turbulence, where η^* is some critical value of the nondimensional shear. The risk R of $\eta > \eta^*$ is equal to $1 - P$. Accordingly, we write

$$P(\eta \leq \eta^*) = 1 - R(\eta > \eta^*) = \int_{-\infty}^{\xi^*} (2\pi)^{-1/2} e^{-\xi^2/2} d\xi \quad (20)$$

where

$$\xi^* = \{[(\Delta V_f)^* - \langle \Delta V_f \rangle]/u^*_0\}/(\sigma/u_{*0}) \quad (21)$$

and $(\Delta V_f)^*$ is the critical value ΔV_f associated with η^*. According to Eq. (20) we may treat ξ^* as a function of P or R and solve Eq. (21) for $(\Delta V_f)^*$ to yield

$$(\Delta V_f)^* = \langle \Delta V_f \rangle + \xi^*(P)\sigma \quad (22)$$

Substitution of Eqs. (11), (12), and (18) into Eq. (22) yields

$$(\Delta V_f)^* = u_{*0}\{k^{-1} \sin\gamma \cos\theta \ln[(1 + \Delta z/2\bar{z})/(1 - \Delta z/2\bar{z})] + 1.41\xi^*[(2.25 \cos^2\theta + 4) \sin^2\gamma + 1.96 \cos^2\gamma - 2 \sin\gamma \cos\gamma \cos\theta]^{1/2}\} \quad (23)$$

where we have set $z/L_0 = 0$ in Eq. (12). Thus, specification of $P(\eta \leq \eta^*)$ or $R(\eta > \eta^*)$, the angles θ and γ, the altitude parameters Δz and $\bar{z}$ and the friction velocity u_{*0} permits the calculation of the shear $(\Delta V_f)^*$ associated with the assigned probability or risk levels for a given realization of turbulence.

During a normal ILS (Instrument Landing System) approach $\gamma \simeq 87°$, so that $\sin\gamma \simeq 1$ and $\cos\gamma \simeq 0$ in Eq. (23). With these approximations it is clear from Eq. (23) that the maximum shear will occur when the mean wind is blowing down the runway ($\theta = 0$). This is the case we shall now consider. To estimate the friction velocity we evaluate Eq. (7) with $z/L_0 = 0$ at some reference anemometer height $z = z_r$, above the zero plane level, which we will take to be 10 m. Thus, we write Eq. (23) in the form

$$(\Delta V_f)^* = [u_r k/\ln(z_r/z_0)]\{\ln[(1 + \Delta z/2\bar{z})/(1 \pm \Delta z/2\bar{z})] + 3.53\xi^*\} \quad (24)$$

where $u_r = \bar{u}(10 \text{ m})$. The wind shears in Table 2 are referenced to a 30 m layer; accordingly, we set $\Delta z = 30$ m in Eq. (24). We now have four parameters available which can be varied, namely, u_r, z_0, $\bar{z}$, and P or R. The effects of these parameters on $(\Delta V_f)^*$ are shown in Figs. 2–4.

Fig. 2 shows $(\Delta V_f)^*$ as a function of u_r for various values of P, $\Delta z = 30$ m, $\bar{z} = 50$ m, and $z_0 = 1$ m. We can see from the

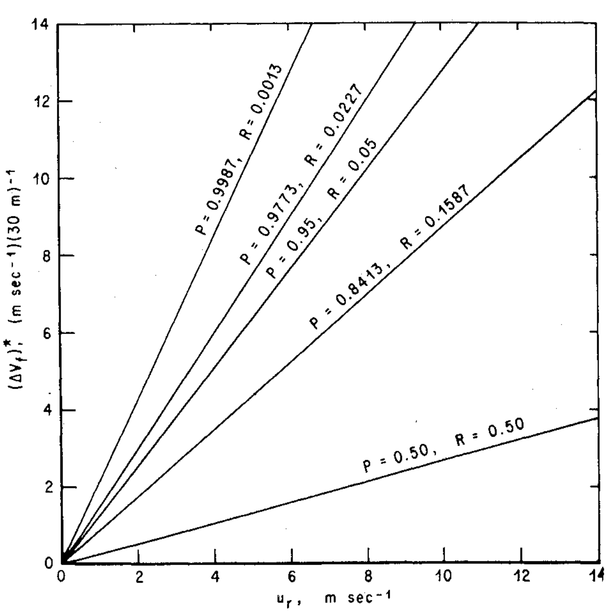

Fig. 2 Wind shear $(\Delta V_f)^*$ as a function of reference level wind speed and risk for $\Delta z = 30$ m, $\bar{z} = 50$ m, and $z_0 = 1$ m.

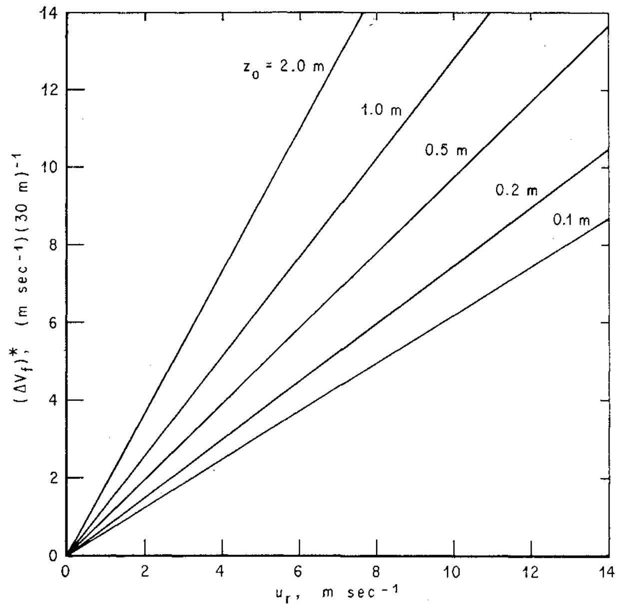

Fig. 3 Wind shear $(\Delta V_f)^*$ as a function of reference level wind speed and surface roughness length for $\Delta z = 30$ m, $\bar{z} = 50$ m, $P = 0.95$ or $R = 0.05$.

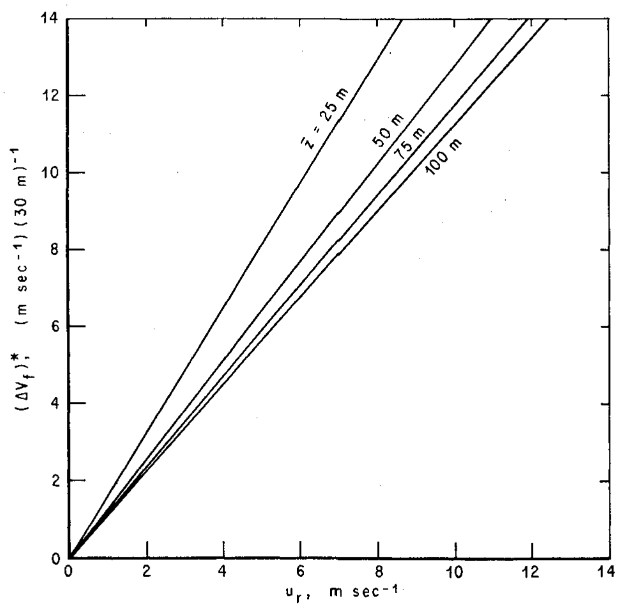

Fig. 4 Wind shear $(\Delta V_f)^*$ as a function of reference level wind speed and shear interval altitude above the zero-plane for $z_0 = 1$ m, $\Delta z = 30$ m, $P = 0.95$ or $R = 0.05$.

plot that for a given value of the mean wind speed at the reference level the dispersion about the mean is relatively large. In this particular case the 95 percentile ($P = 0.95$) shear is approximately equal to 6/5 of the mean wind speed at the reference level. According to the $P = 0.50$ line, moderate shear begins at approximately $u_r = 22$ m sec^{-1}. This can be verified by extending the $R = P = 0.50$ line. Thus, the mean wind profile in the neutrally stable boundary layer produces only light shear for wind speeds in the range $0 < u_r \lesssim 22$ m sec^{-1}. However, when turbulence is included in the analysis we find that the moderate and stronger shear magnitudes can occur for reference level wind speeds $u_r < 22$ m sec^{-1}. For example, according to the $R = 0.1587$ line, moderate shear can occur in the mean wind speed interval $3 \lesssim u_r \lesssim 5.0$ m sec^{-1}, and severe shear occurs at mean wind speeds greater than 7 m sec^{-1}.

The effect of z_0 and $\bar{z}$ on the 95 percentile wind shear is shown in Figs. 3 and 4. Fig. 3 shows $(\Delta V_f)^*$ at the 95 percentile level as a function of u_r for various values of z_0, $\Delta z = 30$ m, and $\bar{z} = 50$ m. A decrease in the roughness length by one order of magnitude produces approximately a 60% decrease in $(\Delta V_f)^*$. This result holds for all values of P. Fig. 4 shows $(\Delta V_f)^*$ at the 95 percentile level as a function of u_r for various values of $\bar{z}$, $\Delta z = 30$ m, and $z_0 = 1$ m. An increase in the mean height of the shear interval from 25 to 100 m will produce approximately a 30% decrease in the 95 percentile shear. It should be kept in mind that variations in $\bar{z}$ merely cause variations in the mean shear without variation in the standard deviation. Thus, a given variation in $\bar{z}$ will produce the same amount of variation in magnitude in all values of $(\Delta V_f)^*$. On the other hand, variations in z_0 produce variation in both the mean and standard deviation of shear. This results because the mean and standard deviation are both directly proportional to u_{*0} which, in turn, is inversely proportional to $\ln(z_r/z_0)$. Wind shears for mean wind directions other than $\theta = 0$ can be obtained by varying θ in Eq. (23). Thus, for example, when the mean wind is 90° to the runway, Eq. (23) reduces to

$$(\Delta V_f)^* = 2.82\, u_{*0}\xi^* \sin\gamma(1 + 0.49 \cot^2\gamma)^{1/2} \qquad (24)$$

Terrain Considerations

Earlier in this paper we assumed the boundary layer to be horizontally homogeneous. Let us consider the nature of this assumption more closely. Most municipal airports in major cities are surrounded by the city or suburbs. The air which flows from the city to the airport will have a turbulence structure which will be characterized by the city roughness length, which is like $z_0 = 1$ to 4 m (see Table 1). However, when the air flows from the rough city to the relatively smooth terrain associated with the airport, the turbulence structure of the air near the ground will be modified by virtue of the fact that the airport will generally have a roughness length one to two orders of magnitude smaller than the city roughness length. Thus, an internal boundary layer will form over the airport. The theoretical development by Townsend and Panofsky[4] predicts the thickness h of this boundary layer will grow downstream from the discontinuity in roughness as $x^{0.8}$ where x is the distance from the change in roughness to the point of concern. Thus, the theory suggests that the new boundary layer will grow with slope of about 1 in 10 downwind from the roughness discontinuity. In terms of aircraft operations, this means that during the let-down flight phase an airplane could pass through two types of boundary-layer flows. The first flow regime would be contolled by the roughness elements of the terrain surrounding the airport. The airplane would then pass through a flowfield interface and then enter the second flow regime controlled by the surface roughness properties of the airport. The reverse order of events would occur for the take-off flight phase. More complicated situations might arise if the distribution of surface roughness surrounding the airport is not horizontally homogeneous. The estimates of wind shear in this paper were obtained for a single flow regime. However, the theory in this paper could be used, as a first approximation, separately in each layer. To estimate the wind shear across the internal boundary layer interface one must re-do the analysis and apply an internal boundary layer theory to the problem. This, perhaps, is calling for too much detail. However, there is one last point to keep in mind. The standard surface wind observations at an airport will be indicative of the city or airport depending on the distance and height of the anemometer from the airport change of roughness. If the mean wind observations (u_r) are controlled by the airport roughness, then these wind observations must be corrected for the change of roughness effect to apply the theory herein to the flow regime above the internal boundary layer. The converse is true if the mean wind observations are controlled by the roughness surrounding the airport. These corrections could be significant.

Conclusions

We have examined the mean and fluctuating portions of wind shear in the neutrally stable boundary layer. Wind shear is a function of 1) the surface roughness length z_0 associated with the terrain surrounding the airport, 2) the mean wind speed u_r at the reference level (height $d + 10$ m above natural grade in the example), 3) the shear height interval Δz, 4) the height $\bar{z}$ of the midpoint of the shear interval above zero plane, and 5) the risk one is willing to accept of encountering the shear magnitude of concern. For the case of an airport located in a city with $z_0 \gtrsim 1$ m and $\bar{z} = 50$ m we found that the mean shear produced by the mean wind speed profile resulted in light shear for mean wind speeds less than approximately 22 m sec^{-1} at the reference level relative to the zero plane. The critical wind speed will vary according to the altitude of the shear interval and the surface roughness length. However, the height of the shear interval was found to have little effect on the mean wind shear for $25 \leq \bar{z} \leq 100$ m. The addition of atmospheric turbulence to the problem results in relatively large statistical dispersions in wind shear about the mean shear. Thus, it is not possible to define a specific wind shear magnitude for the aircraft landing problem without specifying the risk one is willing to accept of encountering a specific wind shear magnitude. The precise value of risk will depend upon the nature of the problem.

This analysis does not close the door on the shear problem. The analysis can be extended to the unstable and stable boundary layers. The extension to unstable and slightly stable boundary layers is straightforward. However, the extension to the extremely stable boundary layer could be somewhat difficult. It was pointed out earlier that for $z/L_0 \gtrsim 0.18$ turbulence ceases. Now turbulence tends to couple the flow between the various levels in the boundary layer. If turbulence ceases, as it often does at night, the levels become uncoupled, so that it becomes extremely difficult to model the behavior of the wind. It should be kept in mind that the night time stable boundary layer could be more dangerous than the daytime fully turbulent boundary layer. This results because the large shears in the daytime boundary layer are the fluctuating shears produced by turbulence, so that if a large shear magnitude occurs, its duration time is relatively short. This means the airplane could fly in and out of the shear condition before the pilot has time to react and the influence on the aircraft would be transitory. However, in the night time boundary layer, the layer of air near the ground cools by radiative heat transfer and a temperature inversion can develop. Below the inversion the boundary layer is stable ($z/L_0 \gtrsim 0.18$) and there is little or no motion of the air. Above the inversion, the air can have relatively high wind speeds. As the airplane descends through the inversion, it can experience a large mean wind shear. In this case the reduction in lift is permanent unless the pilot reacts accordingly.

References

[1] Jones, R. T., "The Unsteady Lift of a Wing of Finite Aspect Ratio," 681, 31-38, 1940, NACA.

[2] Roberts, C. F., "A Preliminary Analysis of Some Observations of Wind Shear in the Lowest 100 Feet of the Atmosphere for Application to the Problem of the Control of Aircraft on Approach," *Vertical Wind Shear in the Lower Layers of the Atmosphere*, TN 93, 1963, World Meteorological Organization, Geneva, Switzerland, pp. 203–218.

[3] Lumley, J. L. and Panofsky, H. A., *The Structure of Atmospheric Turbulence*, Interscience, New York, 1964, pp. 239

[4] Munn, R. E., *Descriptive Micrometeorology*, Academic Press, New York, 1966.

[5] Prasad, B. and Panofsky, H. A., "Properties of Variances of the Meterological Variables at Round Hill," *Properties of Wind and Temperature at Round Hill, South Dartmouth, Mass.*, Final Rept., Grant DAAB07-67-C-0035, 1967, Dept. of Meteorology, The Pennsylvania State Univ., University Park, Pa. pp. 65–92.

[6] "Information for the Approach, Landing, Take-Off and Ground Movement of Aircraft," *Final Report of Fifth Navigation Conference of ICAO*, Doc. 8720, AN-CONF/5, Agenda Item 6, 1967, International Civil Aviation Organization, Montreal, Quebec Canada.

[7] Dutton, J. A., Thompson, G. J., and Deaven, D. G., "The Probabilistic Structure of Clear Air Turbulence—Some Observational Results and Implications," *Clear Air Turbulence and Its Detection*, edited by Yih-Ho Pao and A. Goldburg, Plenum Press New York, 1969, pp. 542

NOVEMBER 1972 J. AIRCRAFT VOL. 9, NO. 11

Application of Geometric Decoupling Theory to Synthesis of Aircraft Lateral Control Systems

EUGENE M. CLIFF* AND FREDERICK H. LUTZE†
Virginia Polytechnic Institute and State University, Blacksburg, Va.

A vector space formulation is used to develop a computational procedure for decoupling a multivariable system. The concepts fundamental to this geometric approach are defined and their relation to the behavior of the linear system is discussed. Two theorems that provide the basis for the procedure are stated. The vector space statements are translated into algebraic operations suitable for machine calculation. A principal result of these computations is a set of linear algebraic equations that the elements of a decoupling feedback matrix must satisfy. The set of all solutions defines (in certain cases) the class of feedback matrices that will give the desired decoupling. A penalty function method is used to choose a matrix from this class so that the augmented system has desirable handling qualities. Two examples from the open literature are used to illustrate the method.

Introduction

IN the crowded terminal areas of today's air transportation system it would be helpful both to provide more precise flight-path control and to reduce pilot workload. These two improvements, which are often conflicting, can be achieved by providing a decoupled flight control system. The core of the decoupling concept is that each control should affect a single output without disturbing others. Pilot workload is reduced since coordination of controls is no longer required. Additionally, flight-path precision can be ehanced since the pilot can focus his attention on a particular output.

The decoupling problem has received considerable attention and several approaches are possible. One method, perhaps the most obvious one, is restricted to the case when the outputs to be decoupled are scalars (e.g., decoupling bank angle from sideslip). The germ of the idea is to find feedback and cross control matrices such that the output/input transfer frunction is diagonal and nonsingular. Using such an approach Falb and Wolovich[1] and Gilbert[2] have established methods of solving this decoupling problem. Falb and Wolovich[1] also present a synthesis procedure which in some cases provides a means of achieving a specified pole-placement while simultaneously decoupling the system. Gilbert and Pivnichny[3] have also reported some computational procedures based on their approach to the synthesis problem. Other computational methods have also been used. Model following techniques were employed by Yore[4] and by Hall.[5] Additionally, Montgomery and Hatch[6] have applied their general synthesis method to the decoupling problem. On the theoretical side an elegant approach, based on a vector space formulation, was developed independently by Wonham and Morse,[7–9] and Basile and Marro.[10, 11] The computational procedures used here are based, in part, on this geometric approach and this is, to our knowledge, the first reported application of this geometric theory.

Our approach to the combined problems of decoupling and handling qualities proceeds in two stages. First we find (in certain cases) the most general class of feedback matrices F and cross control matrices G that lead to a decoupled system. We then use a quadratic penalty function method in conjunction with a numerical descent algorithm to arrive at (or close to) specified handling qualities.

The first step, that of finding the most general F and G matrices, follows the geometric approach of Ref. 9. The theory is outlined in the next section, which includes a simple example. An essential part of the calculation procedure is explained in Appendix A. The following section describes the penalty function method used in specifying handling qualities. We then consider two examples dealing with aircraft lateral control systems. The final section includes a discussion of some difficulties and comparisons with other methods.

Geometric Theory of Decoupling

In order to analyze aircraft motions the standard procedure, which will be followed here, is to linearize the equations of motion about some nominal flight condition. If this condition is a steady motion then the resulting equations are

Received March 14, 1972; revision received July 12, 1972.
Index category: Aircraft Handling, Stability, and Control.

* Assistant Professor, Aerospace Engineering Department. Associate Member AIAA.
† Associate Professor, Aerospace Engineering Department. Associate Member AIAA.